



RM-6573-R

B. E. II (Sem. IV) (Mech.) Examination

May / June - 2010

Maths - III

Time : Hours]

[Total Marks :

Instruction :

(1)

नीचे दर्शाविले निशानीवाणी विगतो उत्तरवही पर अवश्य लपवी.  
 Fillup strictly the details of signs on your answer book.

Name of the Examination :  
 B. E. 2 (Sem. 4) (Mech.)

Name of the Subject :  
 Maths - 3

Subject Code No. : 6 5 7 3 Section No. (1, 2,.....): Nil

Seat No. :

Student's Signature

Q-1 a) Attempt the following [10]

- i) Evaluate  $\int_{\pi/2}^1 \int_1^2 x \cos xy \, dy dx$
- ii) Define periodic function and state the period of  $\sin 2x$ .
- iii) Show that the vector  $\vec{F} = (z + \sin y)i + (x \cos y - z)j + (x - y)k$  is irrotational vector.
- iv) For any closed surface  $\sigma$  show that  $\int_{\sigma} \text{curl } \vec{F} \cdot \hat{n} \, ds = 0$
- v) Show that  $\text{grad}(r) = \frac{\vec{r}}{r}$ . Where,  $\vec{r} = xi + yj + zk$ .

b) Attempt any two of the following. [10]

- i) Evaluate  $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy dx}{\sqrt{1+x^2+y^2}}$
- ii) Evaluate  $\iint_R (2x - y^2) \, dA$  over the triangular region R enclosed by  $y = -x + 1, y = x + 1$  and  $y = 3$ .
- iii) Find the volume bounded by the cylinder  $x^2 + y^2 = 4$  and by the planes  $y + z = 4$  and  $z = 0$ .

Q-2 a) Verify Green's theorem for  $\oint_C [(3x^2 - 8y^2) dx + (4y - 6xy) dy]$  around the boundary C of the region  $x = 0, y = 0$  and  $x + y = 1$ . [6]

b) Attempt any three of the following. [9]

- i) A vector field is given by  $\vec{F} = (\sin y)i + x(1 + \cos y)j$ . Evaluate the line integral  $\int_C \vec{F} \cdot d\vec{r}$  over the circular path  $x^2 + y^2 = a^2$  and  $z = 0$ .

- ii) Show that  $\vec{F} = (6xy + z^3) i + (3x^2 - z) j + (3xz^2 - y) k$  is irrotational & find its scalar potential.
- iii) Find a vector normal to the surface  $x^2y + zx$  at the point  $(1, 0, 2)$  in the direction of  $i - j + 2k$ .
- iv) Find the directional derivative of  $f(x, y, z) = 2xyz + z^2$  at the point  $(1, -1, 3)$  in the direction of the outer normal  $i + 2j + 2k$ .
- Q-3** a) Define periodic function. Obtain Euler's formulae for the periodic function in the interval  $0 < x < 2\pi$ . [5]
- b) **Attempt any two of the following.** [10]
- i) Obtain Fourier series expansion for  $f(x) = x^2, -\pi \leq x \leq \pi$ . Hence show that
- a) 
$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$
- b) 
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2} = \frac{\pi^2}{12}$$
- ii) Obtain Fourier series expansion for the function  $f(x) = x \sin x$  in the interval  $0 < x < 2\pi$ .
- iii) Obtain half-range cosine series for  $f(x) = \begin{cases} kx, & 0 \leq x \leq l/2 \\ k(l-x), & l/2 \leq x \leq l \end{cases}$

### Section-II

- Q-4** a) Attempt the following : [10]
- i) Evaluate  $\int_0^{\pi/2} \cos^{5/2} x dx$
- ii) Prove that  $\operatorname{erf}(x) + \operatorname{erfc}(x) = 1$ .
- iii) Obtain one L.I. solution of the PDE  $zp - zq = z^2 + (x+y)^2$
- iv) Write any two properties of beta function.
- v) Prove that  $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$
- b) **Attempt the following.** [10]
- i) State and prove relation between beta and gamma function. [04]
- ii) Prove that  $\int_0^1 x^m \left( \log \frac{1}{x} \right)^n dx = \frac{n!}{(m+1)^{n+1}}$ . [03]
- iii) Prove that  $\beta(m, n) = \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy$  [03]
- Q-5** a) Solve the following One-dimensional Heat equation [08]
- $$u_t = c^2 u_{xx}; t > 0 \text{ and } 0 < x < l$$
- $$u(0, t) = 0 = u(l, t); t > 0$$
- $$u(x, 0) = f(x)$$
- Using separation of variables method.
- b) **Attempt any one of the following.** [07]
- i) A taut string of length  $l$  has its ends  $x=0$  and  $x=l$  fixed. The midpoint is stretched to a small height and released from rest at time  $t=0$ . Find the deflection  $u(x, t)$ .
- ii) Solve the following boundary value problem
- $$u_{xx} + u_{yy} = 0 \text{ with the conditions}$$
- $$u(0, y) = u(a, y) = 0 \text{ for } 0 \leq y \leq b \text{ and}$$
- $$u(x, 0) = 0 \text{ and } u(x, b) = f(x), 0 \leq x \leq a.$$

- Q-6 a) **Attempt any two of the following.** [06]
- i) Solve  $(y^2 + z^2)p - xyq + xz = 0$
  - ii) Solve  $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$
  - iii) Solve  $xp + yq = x$ .
- b) **Attempt any three of the following.** [09]
- i) A car-hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and proportion of days on which some demand is refused.  
( $e^{-1.5} = 0.2231$ )
  - ii) A sample of 100 battery cells tested to find the length of life produced the following results:  
 $\bar{x} = 12 \text{ hours}, \sigma = 3 \text{ hours}$ .  
Assuming the data to be normally distributed, what percentage of battery cells are expected to have life
    - i. Less than 6 hours
    - ii. More than 15 hours
 [S.N.V. (2)=0.4772 & S.N.V. (1)=0.3413]
  - iii) A random sample of 900 members has a mean 3.4 cms. Can it be reasonably regarded as a sample from a large population of mean 3.2 cms. and standard deviation 2.3 cms.? [z at 5% level of significance is 1.96]
  - iv) A sample of 20 items has mean 42 units and S.D. 5 units. Test the hypothesis it is a random sample from a normal population with mean 45 units. [t at 5% level of significance and 19 d.f. is  $t_{0.05} = 2.09$
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